MATH 590: QUIZ 1 SOLUTIONS

Name:

1. Consider $(\mathbb{R}^+)^2$ with addition $(x_1, x_2) +' (y_1, y_2) := (x_1y_1, x_2y_2)$ and scalar multiplication given by $c * (x_1, x_2) := (x_1^c, x_2^c)$, for $c \in \mathbb{R}$. Verify the two versions of the distributive property for V. (4 points)

Solution. First suppose $c, d \in \mathbb{R}$ and $x, y) \in \mathbb{R}^2$. Then :

$$(c + d) * (x, y) = (x^{c+d}, y^{c+d}) = (x^c x^d, y^c y^d) = (x^c, y^c) +' (x^d, y^d) = c * (x, y) +' d * (x, y).$$

Now suppose $c \in \mathbb{R}$ and $(x_1, y_1), (x_2, y_2) \in \mathbb{R}^2$. Then:

$$\begin{aligned} c * \{(x_1, y_1) +' (x_2, y_2)\} &= c * (x_1x_2, y_2y_2) \\ &= ((x_1x_2)^c, (y_1y_2)^c) \\ &= (x_1^c x_2^c, y_1^c y_2^c) \\ &= (x_1^c, y_1^c) +' (x_2^c, y_2^c) \\ &= c * (x_1, y_1) + \ c * (x_2, y_2). \end{aligned}$$

2. Let V be a vector space over $\mathbb{R}$ and $W \subseteq V$ a subset. Define what it means for W to be a *subspace* of V. (3 points)

Solution. W is a subspace of V if : (i) For all $w_1, w_2 \in W$, $w_1 + w_2 \in W$ and (ii) For all $\lambda \in \mathbb{R}$ and $w \in W$, $\lambda w \in W$.

3. Describe all of the subspaces of $\mathbb{R}^3$.

Solution. The subspaces of $\mathbb{R}^3$ are: $\{\vec{0}\}$, lines through the origin, planes through the origin, and $\mathbb{R}^3$.